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NOTE ON AREAS AND VOLUMES.

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There is so much interest centred around the curve $\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} = 1$, and the surface $\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} + \left(\frac{z}{c}\right)^{\frac{2}{2p+1}} = 1$, that it is necessary to give to the mathematicians a general formula for each that will hold for any positive integral values of m, n, p .

$$A = \text{area} = 4 \int \int dx dy = \frac{4ab}{(2m+1)(2n+1)} \cdot \frac{\Gamma\left(\frac{2m+1}{2}\right) \Gamma\left(\frac{2n+1}{2}\right)}{\Gamma\left(\frac{2m+1}{2} + \frac{2n+1}{2} + 1\right)}$$

$$= \frac{4ab}{\frac{2}{2m+1} + \frac{2}{2n+1}} \cdot \frac{\Gamma\left(\frac{2m+1}{2}\right) \Gamma\left(\frac{2n+1}{2}\right)}{\Gamma\left(\frac{2m+1}{2} + \frac{2n+1}{2}\right)}, \text{ by Dirichlet's theorem under}$$

the above conditions.

$$\therefore A = \frac{ab(2m+1)(2n+1)}{(m+n)(m+n+1)} \cdot \frac{\Gamma(m+\frac{1}{2})\Gamma(n+\frac{1}{2})}{\Gamma(m+n)} \dots (1)$$

$$= \frac{1.3.5 \dots (2m+1) \times 1.3.5 \dots (2n+1)}{2.4.6 \dots 2(m+n+1)} \cdot 2\pi ab.$$

When $m=n=0$, $A=\pi ab$, area of the curve $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$, the ellipse.

When $m=n=1$, $A=\frac{3}{8}\pi ab$, area of the hypocycloid, $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

When $m=0, n=1$, $A=\frac{3}{4}\pi ab$, area of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

When $m=n=2$, $A=\frac{1}{12}\pi ab$, area of the curve $\left(\frac{x}{a}\right)^{\frac{2}{5}} + \left(\frac{y}{b}\right)^{\frac{2}{5}} = 1$.

When $m=1, n=2$, $A=\frac{1}{4}\pi ab$, area of curve $\left(\frac{x}{a}\right)^{\frac{2}{5}} + \left(\frac{y}{b}\right)^{\frac{2}{5}} = 1$.

When $m=n=4$, $A=\frac{1.5.7.9 \cdot \pi ab}{(32)^3}$, area of the curve $\left(\frac{x}{a}\right)^{\frac{2}{9}} + \left(\frac{y}{b}\right)^{\frac{2}{9}} = 1$.

$$V = \text{volume} = 8 \int \int \int dx dy dz$$

$$\begin{aligned}
&= \frac{8abc}{(2m+1)(2n+1)(2p+1)} \cdot \frac{\Gamma\left(\frac{2m+1}{2}\right) \Gamma\left(\frac{2n+1}{2}\right) \Gamma\left(\frac{2p+1}{2}\right)}{\Gamma\left(\frac{2m+1}{2} + \frac{2n+1}{2} + \frac{2p+1}{2} + 1\right)} \\
&= \frac{8abc}{\frac{1}{(2m+1)(2n+1)} + \frac{1}{(2n+1)(2p+1)} + \frac{1}{(2m+1)(2p+1)}} \times \\
&\quad \frac{\Gamma\left(\frac{2m+1}{2}\right) \Gamma\left(\frac{2n+1}{2}\right) \Gamma\left(\frac{2p+1}{2}\right)}{\Gamma\left(\frac{2m+1}{2} + \frac{2n+1}{2} + \frac{2p+1}{2}\right)} \\
&= \frac{4abc(2m+1)(2n+1)(2p+1)}{(2m+2n+2p+3)(2m+2n+2p+1)} \cdot \frac{\Gamma(m+\frac{1}{2})\Gamma(n+\frac{1}{2})\Gamma(p+\frac{1}{2})}{\Gamma(m+n+p+\frac{1}{2})} \dots (2) \\
&= \frac{1.3.5 \dots (2m+1) \times 1.3.5 \dots (2n+1) \times 1.3.5 \dots (2p+1)}{1.3.5 \dots (2m+2n+2p+3)} \cdot 4\pi abc.
\end{aligned}$$

When $m=n=p=0$, $V=\frac{4}{3}\pi abc$, volume of $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$.

When $m=n=p=1$, $V=\frac{4}{3}\pi abc$, volume of $\left(\frac{x}{a}\right)^{\frac{3}{2}} + \left(\frac{y}{b}\right)^{\frac{3}{2}} + \left(\frac{z}{c}\right)^{\frac{3}{2}} = 1$.

When $m=n=p=2$, $V=\frac{4.5}{3.7.11.13}\pi abc$, volume of $\left(\frac{x}{a}\right)^{\frac{5}{2}} + \left(\frac{y}{b}\right)^{\frac{5}{2}} + \left(\frac{z}{c}\right)^{\frac{5}{2}} = 1$.

When $m=n=p=3$, $V=\frac{4.5.7}{9.11.13.17.19}\pi abc$, volume of $\left(\frac{x}{a}\right)^{\frac{7}{2}} + \left(\frac{y}{b}\right)^{\frac{7}{2}} + \left(\frac{z}{c}\right)^{\frac{7}{2}} = 1$.

When $m=0$, $n=1$, $p=2$, $V=\frac{4}{3}\pi abc$, volume of $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^{\frac{3}{2}} + \left(\frac{z}{c}\right)^{\frac{5}{2}} = 1$.

When $m=1$, $n=2$, $p=3$, $V=\frac{4\pi abc}{3.11.13}$, volume of $\left(\frac{x}{a}\right)^{\frac{3}{2}} + \left(\frac{y}{b}\right)^{\frac{5}{2}} + \left(\frac{z}{c}\right)^{\frac{7}{2}} = 1$.

Formulae (1), and (2) will do for any admissible values of m , n , p .

Let $m=n=p=\frac{3}{2}$; then $V = \frac{4abc \times 4.4.4}{12.10} \cdot \frac{[\Gamma(2)]^3}{\Gamma(5)} = \frac{4}{3}abc$,

the volume of $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} + \left(\frac{z}{c}\right)^{\frac{1}{2}} = 1$.

When $m=n=\frac{3}{2}$, $A=ab \cdot \frac{4.4}{4.3} \cdot \frac{[\Gamma(2)]^2}{\Gamma(3)} = \frac{2}{3}ab$, the area of the

curve $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1$.

The above formulae have been expressed in a little different form in the *Mathematical Magazine*, but they are so useful that they will bear repetition here.